Indian Statistical Institute, Bangalore

B. Math. Second Year

Second Semester - Graph Theory

Final Exam — Duration: 3 hours — Date : April 30, 2015

Max Marks: 100

1. Let G be an edge-maximal non-hamiltonian graph on ϑ vertices. Show that $\deg(x) + \deg(y) \leq \vartheta - 1$ for any two non-adjacent vertices x, y of G. [15]

2. For $n \geq 3$, let G_n be the graph whose vertices are n^2 coins arranged in an $n \times n$ square grid. Two vertices are adjacent iff they are on the same row or same column of this grid.

   (a) Show that G_n is strongly regular and compute its parameters.

   (b) Compute the distinct eigenvalues and their multiplicities in the adjacency matrix of G_n. [8+12 = 20]

3. A matchstick graph is a finite graph which may be drawn on the plane in such a way that the vertices are represented by distinct points and the edges are represented by straight line segments of unit length. Show that if any two adjacent vertices of a graph G have two common neighbors then G is not a matchstick graph. [15]

4. Let G be a vertex-minimal imperfect graph with coclique number α and clique number w. Put $n = 1 + \alpha w$. Then show that G has n cocliques $A_1, ..., A_n$ and n cliques $B_1, ..., B_n$ such that, for all $1 \leq i, j \leq n$, $\#(Ai \cap B_j) = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases}$. [20]

5. Let G be a graph with vertex set V. For $A \subseteq V$ let $G \setminus A$ be the induced subgraph of G with vertex set $V \setminus A$, and let $\varepsilon(A)$ denote the number of connected components K of $G \setminus A$ such that $\#(K)$ is odd. If G has a perfect matching (a set of edges partitioning V) then show that $\varepsilon(A) \leq \#(A)$ for all $A \subseteq V$. [15]

6. A Moore graph is a finite connected graph whose girth is one more than twice its diameter. Show that any Moore graph is regular, and find a formula for its number of vertices in terms of its diameter and degree. [15]